Week 9: Gravitational waves

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Exercise 1. *Gravitational waves emitted by a binary system.* Consider a binary system of two stars with the same mass M in orbit around each other. They are separated a distance r from their common center of mass. We treat the motion of the stars in the Newtonian approximation.

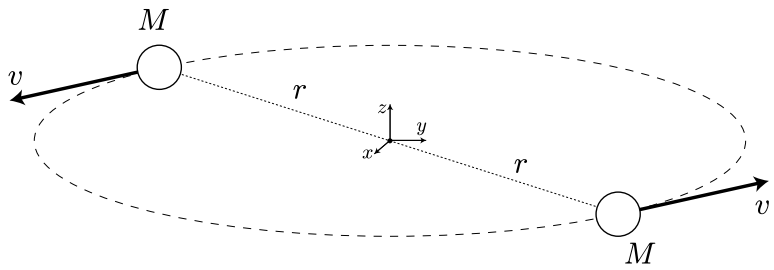

Figure 1: Binary system of equal mass stars.

- Find the velocity v of the stars, the period T and the frequency $\Omega = 2\pi/T$ in terms of G, M and r.
- Choosing the coordinates so that the (x, y)-plane coincides with the equatorial plane of the motion, write down the explicit trajectory

$$\gamma_{(i)}(t) = \Big(x_{(i)}(t), y_{(i)}(t), z_{(i)}(t)\Big) \tag{0.1}$$

  for each of the stars (here, the subindex (i) labels each star, $i = 1, 2$.).
- Argue that the energy density of the system is given by

$$T^{00}(t, \vec{x}) = M \sum_{i=1}^{2} \delta(x - x_{(i)}(t))\delta(y - y_{(i)}(t))\delta(z - z_{(i)}(t)) \,, \tag{0.2}$$

  and find its concrete form using what you found after Eq. (0.1).
- Find the quadropole moment of mass distribution I_{ij} corresponding to T^{00} in Eq. (0.2) and, from it, obtain the components of the metric perturbation in the transverse traceless (TT) gauge.
- Using the gravitational Larmor formula, conclude that the power radiated by the binary is

$$P = \frac{2}{5} \frac{G^4 M^5}{r^5} \,. \tag{0.3}$$