1 The origins of holography

We think of string theory as a theory of quantum gravity. But it was not originally designed to explain spacetime at its fundamental level, nor to be a theory of everything. All that came later—let's see how.

1.1 A glimpse into string theory

In 1959, Tullio Regge realized that the new particles discovered in experiments probing the strong interactions fell into straight lines when their mass squared was plotted as a function of their spin (see Fig. 1). Based on this observation, Veneziano proposed that hadron scattering in the strong interactions could be described by the four-point amplitude

$$A(s,t) = g^2\,\frac{\Gamma(-\,\alpha(s))\,\Gamma(-\,\alpha(t))}{\Gamma(-\,\alpha(s) - \alpha(t))}\,, \tag{1.1}$$

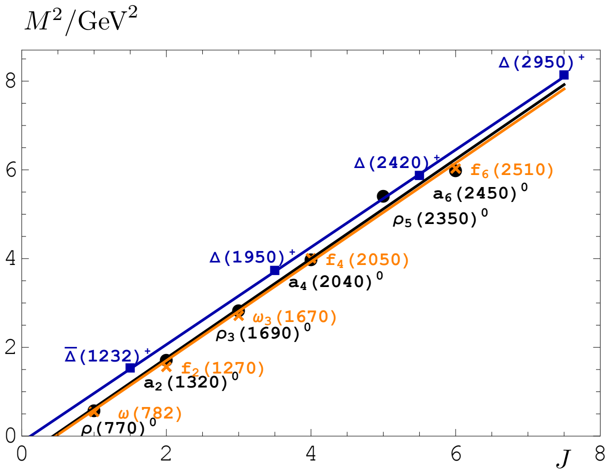

Figure 1: Regge trajectories.

with s, t, and u the Mandelstam variables of the incoming particles, g the coupling constant, and $\alpha(s) = \alpha' s$ a linear Regge trajectory. The amplitude Based on this observation, Veneziano proposed that hadron scattering in the strong interactions was given by the four point amplitude (1.1) was designed to reproduce Regge behavior, satisfy crossing symmetry (treating s, u, and t on equal footing), and produce an infinite tower of resonances.

Remarkably, these ideas led to the development of string theory, because they suggested that the hadrons could be viewed as vibrational states of a relativistic string. To understand this statement, we would like to analyze a rotating relativistic string. But first—how does a relativistic string move?

Recall that the motion of a point-like particle in a given (possibly curved) spacetime is described by a trajectory $X^\mu(\tau)$ that extremizes the length of its *worldline*. In other words, the particle's trajectory is determined by the action

$$S_{\text{particle}} = -m \int \mathrm{d}\tau \sqrt{-G_{\mu\nu}\frac{\mathrm{d}X^\mu}{\mathrm{d}\tau}\frac{\mathrm{d}X^\nu}{\mathrm{d}\tau}}\,. \tag{1.1}$$

where m is the mass of the particle, $G_{\mu\nu}(X)$ is the spacetime metric,

$$\mathrm{d}s^2 = G_{\mu\nu}\,\mathrm{d}X^\mu \mathrm{d}X^\nu. \tag{1.2}$$

A string, on the other hand, extends over a two-dimensional surface in spacetime —its *worldsheet* (see Fig. 2)— and it is natural to require that the motion of the string extremizes the area of this surface. Typically, we parametrize the worldsheet by one timelike coordinate τ and one spacelike coordinate σ (often denoting $(\sigma^1, \sigma^2) = (\tau, \sigma)$). Hence, the action for a free relativistic string is

$$S_{\text{NG}} = -T_s \int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{-\det\left(G_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta}\right)}\,, \tag{1.3}$$

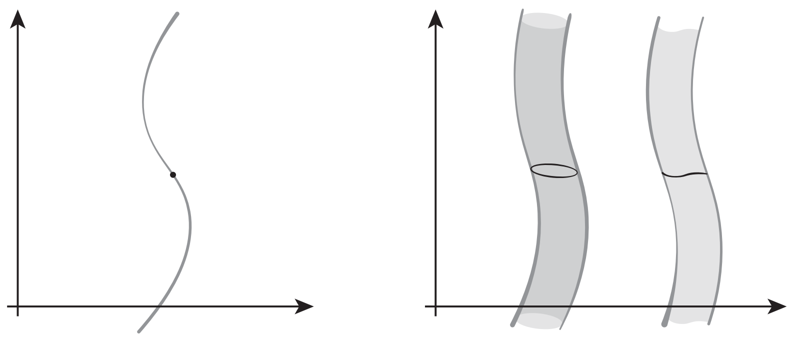

Figure 2: (Left) Worldline of a point-like particle propagating in spacetime. (Right) Worldsheet of closed and open string-like particles propagating in spacetime. In both pictures, the vertical and horizontal axes represent the flow of time and the spacial directions respectively.

where T_s is the string tension. This is the Nambu–Goto (NG) action. The string can be closed or open, depending on whether σ is periodic or not.

Consider now a rotating string in four-dimensional flat space. In this case the metric is just Minkowski,

$$\mathrm{d}s_4^2 = G_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\,. \tag{1.4}$$

Because of the symmetries of the problem, a straight open string lying in the $z = 0$ plane is a solution of the equations of motion. We can parameterize such a string as:

$$\tau = t, \qquad \sigma = r, \qquad x = r\cos(\theta(\tau)), \qquad y = r\sin(\theta(\tau)), \qquad z = 0\,, \tag{1.5}$$

with $\sigma \in (-l,\, l)$ so that the string's total length is $2l$. Substituting this ansatz into Eq. (1.3) we get

$$S_{\mathrm{NG}} = -T_s \int \mathrm{d}t\mathrm{d}r\sqrt{1 - r^2\theta'(t)}\,. \tag{1.6}$$

Because the action does not depend explicitly on $\theta(t)$, the associated conjugate momentum is conserved. Taking $\theta(t) = \omega t$, the conserved quantity is the *spin* (angular momentum) of the string:

$$J = \frac{\partial \mathcal{L}}{\partial \theta'(t)} = T_s \int_{-l}^{l} \frac{\omega r^2}{\sqrt{1 - r^2\omega^2}}\mathrm{d}r = T_s \frac{\pi}{2\omega^2} \tag{1.7}$$

where in the last step we have imposed that the endpoints of the string are massless, so that $l\,\omega = 1$ (the endpoints must move at the speed of light).

Moreover, the action does not depend explicitly on time, which means the energy is conserved. We find

$$E = \theta'(t)\frac{\partial \mathcal{L}}{\partial \theta'(t)} - \mathcal{L} = T_s \int_{-l}^{l} \frac{\mathrm{d}r}{\sqrt{1 - r^2\omega^2}} = T_s \pi l = T_s \frac{\pi}{\omega}\,, \tag{1.8}$$

using $l\,\omega = 1$ in the last equality. This result contains two important bits of information. On the one hand, it gives us the interpretation of T_s as the energy per unit length of the string (indeed, $E = T_s \times 2l$). It also tells us that the energy of a relativistic string grows linearly with its length (which is not the case for an ordinary classical string). In addition, Eq. (1.8) allows us to find a relation between the energy and the spin of the string ,

$$J = \frac{E^2}{2\pi T_s} \equiv \alpha' E^2\,. \tag{1.9}$$

This is nothing but the Regge behavior shown in Fig. (1). Note that the string tension is often expressed in terms of the Regge slope α':

$$T_s = \frac{1}{2\pi\alpha'}\,. \tag{1.10}$$

The tension T_s has thus dimensions of (energy)^2, and for this reason it is also common to define the *string length scale* ℓ_s via $\ell_s^2 \equiv \alpha'$.

1.2 A quantum theory for relativistic strings

We have now seen why a theory of relativistic strings looked promising as a model for the strong interactions. However, hadron scattering and resonance physics are inherently quantum phenomena, so we need to take a step further and construct a quantum theory of relativistic strings. This is how modern string theory was born.

Quantizing the action (1.3). is a complicated and lengthy task (and is part of the content of the String Theory course). Here we will only summarize the main steps and state the results that we will require later. Details can be found in Ref. [1].

The first observation is that Eq. (1.3) can be understood as a two-dimensional field theory for D scalar fields $X^\mu(\sigma)$ defined on the string worldsheet. We will assume the string propagates in flat spacetime, hence we set $G_{\mu\nu} = \eta_{\mu\nu}$. Unfortunately, the Nambu–Goto action is highly non-linear due to the square root. A major insight is that the string action can be rewritten in an equivalent form without the square root, at the expense of introducing an auxiliary field $g_{ab}(\sigma)$ (the worldsheet metric). The result is known as the Polyakov action

$$S_{\rm P} = -\frac{T}{2}\int \mathrm{d}^2\sigma\,\sqrt{-g}\,g^{ab}\,\partial_a X^\mu \partial_b X^\nu\,\eta_{\mu\nu}\,, \tag{1.11}$$

with $g = \det g_{ab}$. This version of the string action eliminates the square root at the expense of introducing an additional field g_{ab}, the wordlsheet metric, and is called the Polyakov action.

The Polyakov action has the following symmetries at the classical level:

- **Target-space Poincaré invariance** (a global symmetry on the worldsheet):

  $$X^\mu \mapsto \tilde{X}^\nu = \Lambda^\mu{}_\nu X^\nu + c^\mu\,, \tag{1.12}$$

  with $\Lambda^\mu{}_\nu$ a constant Lorentz transformation and c^μ a constant translation in the target spacetime.

- **Worldsheet reparametrization invariance** (a gauge symmetry on the worldsheet): under a change of worldsheet coordinates $\sigma^a \to \tilde{\sigma}^a(\sigma)$, the fields transform as

  $$X^\mu(\sigma) \to \tilde{X}^\mu(\tilde{\sigma}) = X^\mu(\sigma)\,, \qquad g_{\alpha\beta}(\sigma) \to \tilde{g}_{\alpha\beta}(\tilde{\sigma}) = \frac{\partial\sigma^\gamma}{\partial\tilde{\sigma}^\alpha}\frac{\partial\sigma^\delta}{\partial\tilde{\sigma}^\beta} g_{\gamma\delta}(\sigma)\,. \tag{1.13}$$

- **Weyl invariance** (another gauge symmetry on the worldsheet): a local rescaling of the metric,

  $$X^\mu(\sigma) \to \tilde{X}^\mu(\sigma) = X^\mu(\sigma)\,, \qquad g_{\alpha\beta}(\sigma) \to \tilde{g}_{\alpha\beta}(\sigma) = \Omega(\sigma)^2 g_{\alpha\beta}(\sigma)\,, \tag{1.14}$$

  which leaves the Polyakov action (1.11) invariant. This symmetry can be thought of as a local change of scale on the worldsheet that preserves angles (i.e. a local conformal symmetry).

The crucial point is that these symmetries can also be realized at the quantum level. In other words, Eq. (1.11) defines a two-dimensional conformal field theory (CFT) of D free scalar fields. However, maintaining all these symmetries in the quantum theory imposes constraints on the theory itself. Famously, requiring quantum Weyl invariance (vanishing of the conformal anomaly) fixes the number of spacetime dimensions — which from the worldsheet point of view is the number of X^μ fields — to be $D = 26$.

Once these consistency conditions are imposed, we can ask what the spectrum of the theory looks like by analyzing the quantum vibrations of the string. It turns out that the various excitations of the string give rise to a tower of particle states whose masses are inversely proportional to the string length. The first of these states is unfortunately a *tachyon* (an excitation with negative mass-squared), which signals an instability of the naive vacuum. The tachyon appears in the spectrum of both open and closed bosonic strings.

If we ignore the tachyon, what other states does the string have? We find a whole zoo of massless states, which fall into three irreducible representations — each associated with a massless field in spacetime:

- A traceless symmetric tensor $G_{\mu\nu}(X)$. This state is the most important of all, since it is a massless spin-2 particle (a graviton). We therefore identify $G_{\mu\nu}$ as the spacetime metric (gravitational field).
- A traceless antisymmetric 2-form $B_{\mu\nu}(X)$, often called the Kalb–Ramond field.
- A scalar (the trace part) $\Phi(X)$, called the dilaton. The asymptotic value of the dilaton field is related to the string coupling constant introduced earlier: $g_s = e^{\Phi(\infty)}$.

These three massless fields are common to all string theories. Apart from them, there is an infinite tower of massive excited states whose masses are on the order of the string scale:

$$M_n \simeq n\ell_s^{-1}\,, \qquad n \in \mathbb{Z}^+\,. \tag{1.15}$$

If the string length ℓ_s is sufficiently small, we do not expect to see these heavy states in a low-energy experiment. Thus, at low energies we are left only with the massless fields listed above.

One can eliminate the tachyon by introducing fermionic degrees of freedom on the worldsheet and imposing supersymmetry. This leads to the *superstring*. In fact, after doing so we are no longer left with a unique low-energy description, but rather several different ones — each corresponding to a ten-dimensional *supergravity* theory. In these lectures we will focus on Type IIB string theory, which is a theory of closed, oriented strings. In addition to the massless fields introduced above, Type IIB has an extra scalar C_0, a 2-form C_2, and a 4-form C_4, which are gauge potentials The low-energy effective theory is a chiral $\mathcal{N} = 2$, $D = 10$ supergravity. The bosonic part of the Type IIB supergravity action is

$$\begin{aligned} S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}X \Bigg[&\sqrt{-G}\,\Big[e^{-2\phi}\Big(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{12}H^2\Big) - \frac{1}{2}\Big(F_1^2 + \frac{1}{3!}F_3^2 + \frac{1}{2}\frac{1}{5!}F_5^2\Big)\Big] \\ &- \frac{1}{2}C_4 \wedge H \wedge \mathrm{d}C_2 \Bigg]\,, \end{aligned} \tag{1.16}$$

where $H = \mathrm{d}B$, $F_1 = \mathrm{d}C_0$, $F_3 = \mathrm{d}C_2 - C_0 H$ and $F_5 = \mathrm{d}C_4 - H \wedge C_2$.

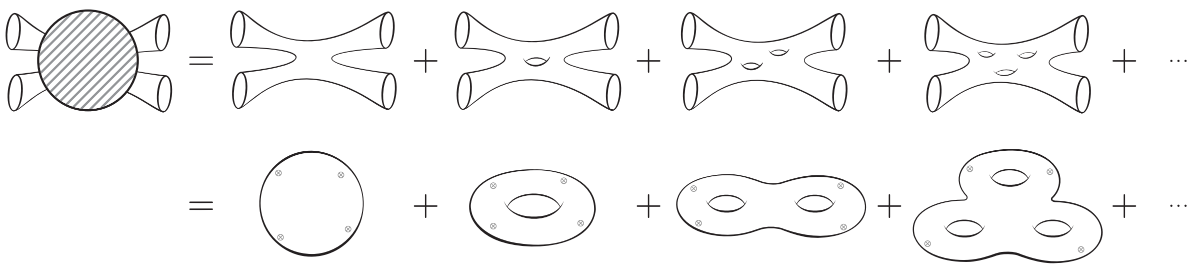

Figure 3: String amplitude as a genus expansion. Each manifold connecting the asymptotic states that are scattered (top) becomes, through a conformal transformation, a compact two dimensional surface with insertion of vertex operators —the gray "⊗" in the picture— that correspond to each state at infinity (bottom). The corresponding values for the Euler characteristic $\chi = 2 - 2\mathsf{g}$ is also indicated below each surface, from which it is clear that we can identify the genus g with the number of "handles" of each manifold.

1.3 String interactions

String interactions can be introduced by adding an extra term to the Polyakov action. In what follows we work in Euclidean signature on the worldsheet. The additional term is also invariant under worldsheet reparameterizations and Weyl transformations, and is given by

$$S_{\text{int}} = \frac{c}{4\pi} \int \mathrm{d}^2\sigma \sqrt{g} R \equiv c\chi \,. \tag{1.17}$$

This looks like an Einstein–Hilbert term on the worldsheet, and one might think it induces 2D gravity on the worldsheet. But this is not the case, because Eq. (1.17) iis actually proportional to the Euler characteristic of the worldsheet, which is a topological invariant (and thus does not affect the equations of motion). More precisely, for a compact orientable surface the Euler characteristic is an integer determined by its genus g,

$$\chi = 2 - 2\mathsf{g} \,, \tag{1.18}$$

The scattering amplitude in string theory, obtained by integrating over all worldsheet metrics, thus becomes a sum over topologies. More explicitly, the n-particle scattering amplitude obtained from the path integral is

$$\begin{aligned} \mathcal{A}^{(n)}(p_1, \cdots, p_n) &= \int \mathcal{D}X \, \mathcal{D}g \, e^{-S_{\text{P}} - S_{\text{int}}} \prod_{i=1}^{n} V_i(p_i) \\ &= \sum_{\text{topologies}} g_s^{-\chi} \int \mathcal{D}X \, \mathcal{D}g \, e^{-S_{\text{P}}} \prod_{i=1}^{n} V_i(p_i) \equiv \sum_{\text{topologies}} g_s^{-\chi} f_\chi(\ell_s) \,. \end{aligned} \tag{1.19}$$

This is illustrated in Fig. 3. Here $V_i(p_i)$ are vertex operators of the 2D CFT inserted on the worldsheet to represent the external states, and $f_\chi(\ell_s)$ is some function that depends on the topology (Euler characteristic) of the worldsheet as well as on the string length scale ℓ_s. In turn, this function can itself be expanded in powers of ℓ_s (assuming ℓ_s is small compared to the other relevant scales),

$$\mathcal{A}^{(n)}(p_1, \cdots, p_n) = \sum_{\text{topologies}} g_s^{-\chi} \sum_{k=0}^{\infty} c_{\chi,k} \ell_s^k \tag{1.20}$$

Using this formalism, one can study the scattering of string states. For example, it turns out that the four-point tachyon scattering amplitude in open string theory coincides *exactly* with the Veneziano amplitude (1.1). This result generated a lot of excitement. However, further experiments revealed that the Veneziano amplitude failed to explain some of the new observations, and string theory was set aside as a model for strong interactions at the same time that Quantum Chromodynamics (QCD) gained prominence due to its success in explaining the data.

1.4 The large-N limit of gauge theories

We mentioned in the previous section that string theory ceased to be considered a viable theory of strong interactions after the success of QCD. Still, it is famously difficult to explain certain infrared phenomena —such as the linear Regge trajectories— using QCD alone, due to its property of asymptotic freedom.

Recall that the QCD Lagrangian consists of two parts: the Yang–Mills term and the fermionic term,

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{fermions}} = -\frac{1}{4}\, F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f . \tag{1.21}$$

Here the sum runs over the different fermion species f, and the non-Abelian field strength is given in terms of the gluon fields A^a_μ in the usual way:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i g_{\text{YM}} f^{abc} A^b_\mu A^c_\nu \,, \tag{1.22}$$

with g_{YM} the Yang-Mills coupling.

In QCD the gauge group is $SU(3)$, and there are 6 quark flavors. For reasons that will become clear later, we generalize this theory by replacing the gauge group with $SU(N)$. We will also consider n_f fermion flavors, although they will not play an important role in the discussion that follows. The index a on the gluon field now runs over $N^2 - 1$ values (the number of generators of $SU(N)$), which is the number of degrees of freedom in the gluonic sector. The quarks, on the other hand, transform in the fundamental representation of the color group, so there are a total of $n_f N$ degrees of freedom in the fermionic sector. Importantly, we will be interested in taking the limit $N \to \infty$, in which case the gluonic degrees of freedom dominate.

At one loop, the Yang–Mills coupling g_{YM} gets renormalized. Remarkably, it has the opposite behavior than the electromagnetic coupling in QED: g_{YM} decreases towards high energies (the UV) and increases at low energies (the IR). This behavior is encapsulated by the beta function, which encodes the running of the coupling with the energy scale μ:

$$\beta_{g^2_{\text{YM}}} \equiv \frac{\mathrm{d} g^2_{\text{YM}}}{\mathrm{d}\log \mu^2} = \mu^2 \frac{\mathrm{d} g^2_{\text{YM}}}{\mathrm{d}\mu^2} = -\beta_0 g^4_{\text{YM}} \,, \qquad \beta_0 = \frac{11N - 2n_f}{48\pi^2} \,. \tag{1.23}$$

Notice that the term which makes the beta function negative (thereby giving asymptotic freedom) scales linearly with N in the large-N limit. The reason is that this term comes from the one-loop contribution to the gluon self-energy shown in Fig. 4. That diagram has two vertices and a single color index running in the loop, hence it scales as $g^2_{\text{YM}} N$. If we want this quantity to remain finite as $N \to \infty$, we need to have $g_{\text{YM}} \sim 1/N$. In other words, we define the *'t Hooft coupling*

$$\lambda \equiv g^2_{\text{YM}} N \,, \tag{1.24}$$

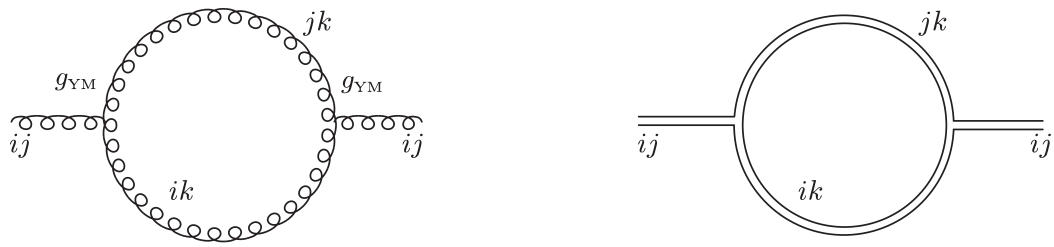

Figure 4: (Left) One-gluon-loop contribution to the gluon propagator. (Right) Same diagram in the double line notation.

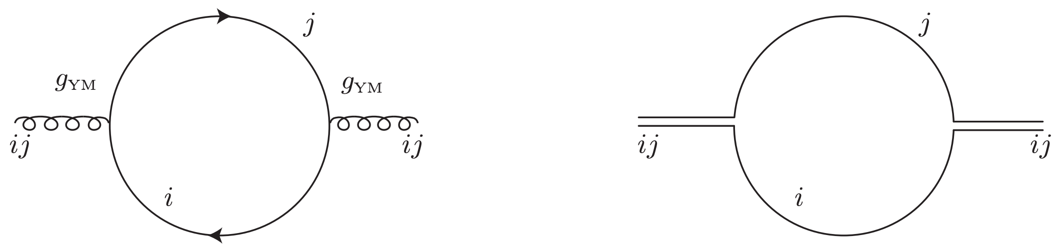

Figure 5: (Left) Quark loop contributing to the gluon propagator. (Right) Same diagram in the double line notation.

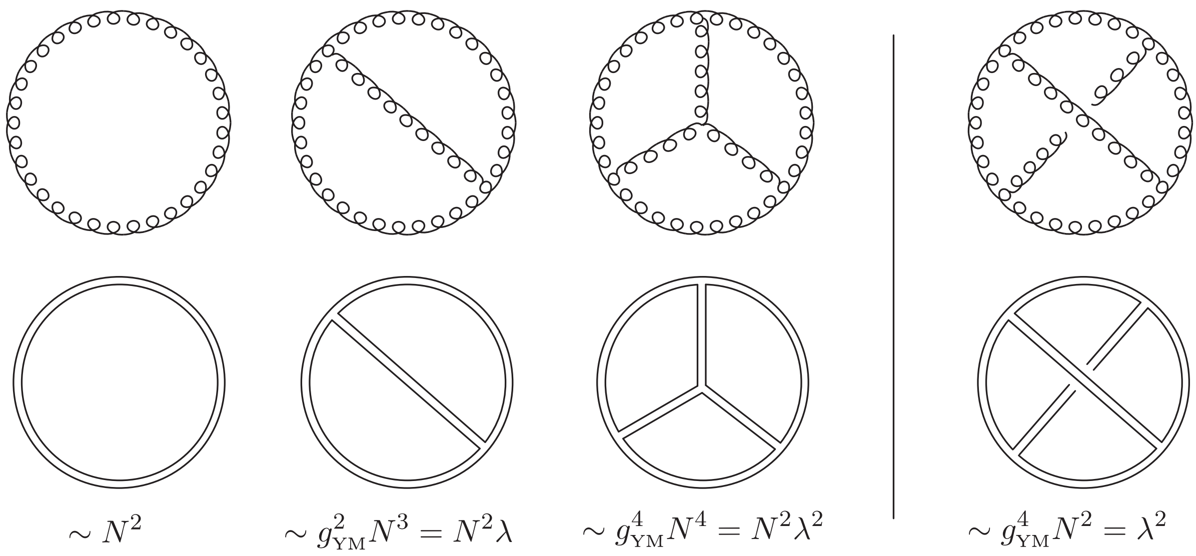

Figure 6: Different scaling with N and λ of different vacuum diagrams.

and consider the limit $N \to \infty$ with λ held fixed. Note that the beta function for this new coupling is

$$\beta_\lambda = \frac{\mu}{2}\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} \propto -\lambda^2, \tag{1.25}$$

which no longer scales with N.

To make the most of this redefinition, it is useful to introduce the *double-line* notation for Feynman diagrams, where each color index is drawn as a separate line. In this notation, gluons are represented by two lines (see Fig. 4 (right)) while quarks are depicted by a single line (see Fig. 5 (right)).

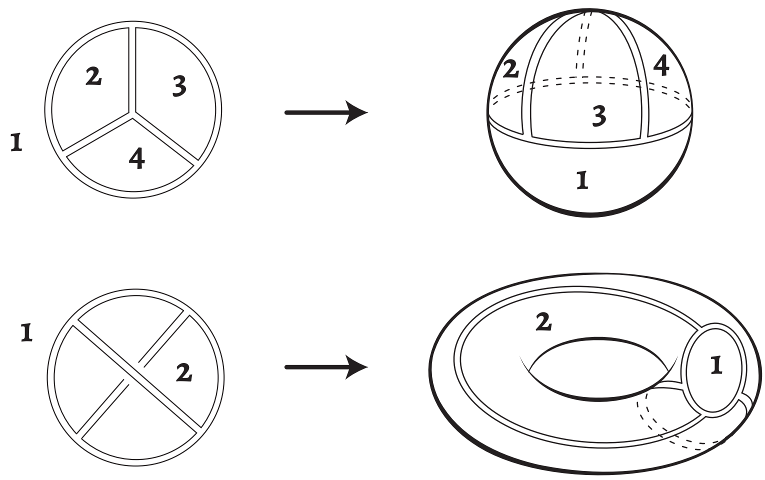

Figure 7: Diagrams are sorted by topology.

Using the double-line notation, we can observe that Feynman diagrams organize themselves in a double expansion in $1/N$ and λ. To understand this, consider vacuum diagrams made out of gluon loops, as in Fig. 6. The scaling with λ is determined by the number of loops l: in fact, a diagram with l loops scales as λ^{l-1}. On the other hand, the scaling with N is determined by the topology of the diagram: planar diagrams (those that can be drawn on a sphere with no gluon lines crossing) scale as $\sim N^2$, whereas diagrams with handles (or other non-planar features) are suppressed by powers of N.

More precisely, one can associate to every Feynman diagram a two-dimensional surface (a Riemann surface), as shown in Fig. 7. The power of N associated to a given diagram is N^χ, where χ is the Euler characteristic of the corresponding surface. Then, any gauge theory amplitude can be expanded in the form

$$\mathcal{A} = \sum_{\mathrm{g}=0}^{\infty} N^{2-2\mathrm{g}} \sum_{n=0}^{\infty} c_{\mathrm{g},n} \lambda^n \,. \tag{1.26}$$

This result is remarkable: we recognize the structure of the string theory amplitude (1.19), if we identify $g_s \sim N^{-1}$. This suggests interpreting the large-N expansion of gauge theories as a perturbative expansion in a dual string theory. Moreover, one expects the 't Hooft coupling λ to be related to the expansion in powers of ℓ_s on the string side.

For completeness, let us mention that adding n_f quark flavors would correspond to inserting n_f boundaries on the Riemann surface; in general, for a surface with b boundaries the Euler characteristic is $\chi = 2 - 2\mathrm{g} - b$, with b the number of boundaries. See Ref. [2] for further details.

1.5 Why would gravity be holographic

From the discussion above, we have uncovered a suggestive connection between theories containing gravity (like string theory) and gauge theories (like large-N QCD), via the large-N expansion and the genus expansion of string amplitudes. Independently of this development, several arguments in black hole physics led to the conclusion that gravity itself should have a *holographic* nature. Both lines of thought will converge in the next section, where we introduce the AdS/CFT correspondence.

The work of Bekenstein and Hawking in the 1970s led to the realization that black holes are thermodynamic objects, characterized by a temperature proportional to their surface gravity and an entropy proportional to the area of their event horizon. In particular, the Bekenstein–Hawking entropy is given by

$$S_{\text{BH}} \sim \frac{A}{\ell_p^2}\,, \tag{1.27}$$

where A is the horizon area and ℓ_p is the Planck length. This result is highly counter-intuitive, as it suggests that the number of fundamental degrees of freedom associated with a region of spacetime scales with its *area*, rather than its volume.

To see why this is surprising, consider a system of n independent quantum bits contained within a spatial region of volume V. The number of distinct orthogonal states of this system is

$$\mathsf{N}(V) = 2^n\,, \tag{1.28}$$

and the corresponding entropy is

$$S = \log \mathsf{N}(V) = n \log 2 = V \ell_p{}^{-3} \log 2 \sim V\,, \tag{1.29}$$

where we have assumed one bit per Planck volume. This naive counting suggests that entropy should scale extensively with the volume.

The crucial observation is that many of these states would carry such a large energy density that they would inevitably collapse into a black hole. In fact, one can argue that (at fixed energy) a black hole maximizes the entropy that can be contained within a given region of space[3–5]. If a hypothetical configuration had higher entropy than the corresponding black hole, one could add matter to induce gravitational collapse into a black hole with lower entropy, thereby violating the second law of thermodynamics.

A complementary argument, following Ref. [3], considers a gas of massless particles in a volume V. On dimensional grounds, its energy and entropy would scale as

$$E \sim VT^4\,, \qquad S \sim VT^3\,. \tag{1.30}$$

However, the energy cannot be arbitrarily large, since once it exceeds a critical value on the order of the region's size, gravitational collapse will occur. The maximum energy E_* is determined by the requirement that the Schwarzschild radius r_s associated with this energy does not exceed the size of the region. This condition is roughly

$$r_s \sim V^{1/3} \sim E_* \ell_p^2\,. \tag{1.31}$$

This implies an upper bound T_* on the temperature,

$$T_* \sim V^{-1/6} \ell_p^{-1/2}\,, \tag{1.32}$$

and consequently an upper bound on the entropy,

$$S_* \sim V\,T_*^3 \sim V^{1/2} \ell_p^{-3/2} \sim \left(\frac{A}{\ell_p^2}\right)^{3/4}\,, \tag{1.33}$$

up to dimensionless constants. For sufficiently large regions, this bound is parametrically smaller than the entropy of a black hole occupying the same volume, which scales as $S_{\text{BH}} \sim A/\ell_p^2$.

These considerations lead to a striking conclusion: the maximum entropy that can be stored within any region of space scales with the area of its boundary, not with its volume. This suggests that the fundamental degrees of freedom of quantum gravity inside a region can be captured by a theory living on the boundary of that region, i.e. in one fewer spatial dimension. This idea is known as the **holographic principle**.

Recommended readings

- Sections 1 and 2 of Ref. [2].
- Section 4.2.1 from Ref. [2].
- Pages 2 and 3 from Ref. [4].

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